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Computing Topological Indices of Honeycomb Derived Networks

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Abstract. A topological index can be considered as a transformation of a chemical structure into a real number. The degree based topological indices such as Randić index, geometric-arithmetic (GA) index and atom-bond connectivity (ABC) index are of vital importance among all topological indices. These topological descriptors significantly correlate certain physico-chemical properties of the corresponding chemical compounds. Graph theory has been found to be very useful in this area of research.
The topological indices of certain interconnection and mesh derived networks are recently studied by Imran et al. [17]. In this paper, we define some new classes of networks from honeycomb networks by using basic graph operations like stellation, medial and dual of a graph. We derive analytical close formulas of general Randić index $R_\alpha(G)$ (for different values of α) for hexagonal and honeycomb derived networks. We also compute first Zagreb, ABC, and GA indices for these important classes of networks.

Key-words: General Randić index, First Zagreb index, Atom-bond connectivity (ABC) index, Geometric-arithmetic (GA) index, Honeycomb derived networks.

2010 Mathematics Subject Classification: 05C12, 05C90

1. Introduction and preliminary results

A *topological index* is a function "Top" defined from $\sum$ to the set of real numbers, where $\sum$ is the set of all finite simple graphs such that for $G, H \in \sum$, we have $Top(G) = Top(H)$ if G and H are isomorphic. A topological index is a numeric quantity associated with the chemical constitution of a chemical compound aiming the correlation of chemical structure with many of its physico-chemical properties, chemical reactivity or biological activities. Topological indices are designed on the ground of transformation of a molecular graph into a number which characterize the topology of that graph.

Motivated by the chemical significance of topological indices, a lot of research has been done in this area and different graph families have been studied. For example, a fixed interconnection parallel architecture is characterized by a graph, with vertices corresponding to processing nodes and edges representing communication links. Interconnection networks are notoriously hard to compare in abstract terms. Researchers in parallel processing are thus motivated to propose new and improved interconnection networks, arguing the benefits and offering performance evaluations in different contexts [6, 33]. A few networks such as hexagonal, honeycomb, and grid networks, for instance, bear resemblance to atomic or molecular lattice structures. These networks have very interesting topological properties which have been studied in different aspects in [1, 2, 7, 12, 17, 22, 26].

The hexagonal and honeycomb networks have also been recognized as crucial for evolutionary biology, in particular for the evolution of cooperation, where the overlapping triangles are vital for the propagation of cooperation in social dilemmas. Relevant research that applies this theory and which could benefit further from the insights of the new research is found in [24, 25, 29, 32].

A graph $G(V, E)$ with vertex set V and edge set E is said to be connected, if there exist a connection between any pair of vertices in G. A *network* is a connected graph having no multiple edges and loops. A *planar graph*, usually represented as $G(V, E, F)$, where F is the set of all regions (or faces), is a graph which can be drawn in the plane without crossing any edge, and such a drawing of G is called its plane drawing. A *chemical graph* is a graph whose vertices denote atoms and edges denote chemical bonds between atoms of the underlying chemical structure.

A (u, v)-path of length l in G is a sequence of $l + 1$ vertices and l edges, from u to v, where $u, v \in V$. The *degree* d_u of a vertex $u \in V$ is the number of vertices which are connected to u by an edge. In a chemical graph the degree of any vertex is atmost 4. The *distance* between two vertices u and v is denoted as $d(u, v)$ and is the shortest path between u and v in G. The length of shortest path between u and v is also called $u - v$ *geodesic*. The longest path between any two vertices $u, v \in V$, is called $u - v$ detour.

Throughout this paper, G is considered to be a network with vertex set $V(G)$, edge set $E(G)$ and d_u is the degree of vertex $u \in V(G)$. The notations used in this paper are mainly taken from the books [8, 10].

The theory of topological indices has its origin in the work done by *Harold Wiener* in 1947 while he was working on boiling point of paraffin. He named this index as *path number*. Later on, the term "path number" was renamed as *Wiener index* [31].

Definition 1.1. Let G be a graph. Then *the Wiener index of G* is defined as

$$W(G) = \frac{1}{2} \sum_{(u,v)} d(u, v),$$

where (u, v) is any ordered pair of vertices in G and $d(u, v)$ is the length of an $u - v$ geodesic. One of the oldest degree based topological index is *Randić* index [27] which was introduced by *Milan Randić* in 1975.

Definition 1.2. *The Randić index of graph G* is defined as

$$R_{-\frac{1}{2}}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}}.$$

The general Randić index was proposed by Bollobás and Erdös [4] and Amic et al. [3] independently, in 1998. Since then it has been extensively studied by both mathematicians and theoretical chemists [16]. Many important mathematical properties have been established [5] and a survey of results can be found in [20].
The general *Randić* index $R_\alpha(G)$ is the sum of $(d_u d_v)^\alpha$ over all edges $e = uv \in E(G)$ defined as

$$R_\alpha(G) = \sum_{uv \in E(G)} (d_u d_v)^\alpha. \quad (1)$$

It can be seen that $R_{-\frac{1}{2}}(G)$ is the particular case of $R_\alpha(G)$, when $\alpha = -\frac{1}{2}$.
An important topological index introduced about forty years ago by *Gutman* and *Trinajstić* [11] is the *Zagreb index* or more precisely first Zagreb index, defined as follows.

Definition 1.3. Consider a graph G, then *first Zagreb index* is defined as

$$M_1(G) = \sum_{uv \in E(G)} (d_u + d_v). \quad (2)$$

The second Zagreb index is defined in the following way.

Definition 1.4. Consider a graph G, then *second Zagreb index* is defined as

$$M_2(G) = \sum_{uv \in E(G)} (d_u \times d_v). \quad (3)$$

One of the well-known degree based topological indices is the *atom-bond connectivity* (*ABC*) index introduced by *Estrada* et al. in [9].

Definition 1.5. For a graph G, *the ABC index* is defined as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}. \tag{4}$$

Another well-known connectivity topological descriptor is *geometric-arithmetic* (GA) index which was introduced by *Vukičević* et al. in [30].

Definition 1.6. Consider a graph G, then its *GA index* is defined as

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{(d_u + d_v)}. \tag{5}$$

2. Main results

In this paper, we construct some new structures derived from hexagonal and honeycomb networks by using some basic graph operations. We give closed analytic formulas of the general Randić $R_\alpha(G)$, first Zagreb $M_1(G)$, atom-bond connectivity $ABC(G)$ and geometric-arithmetic $GA(G)$ indices for these hexagonal and honeycomb derived networks. For further study of topological indices of various graph families see [13–15, 19, 21].

2.1. Honeycomb derived networks

There are numerous open problems suggested for various interconnection networks. To quote Stojmenovic [28]:

> *Designing new architectures remains an area of intensive investigation given that there is no clear winner among existing ones.*

In hexagonal network $HX(n)$, the parameter n is the number of vertices on each side of the network, whereas for honeycomb network $HC(n)$, n is the number of hexagons between boundary and central hexagon. Topological indices of hexagonal, honeycomb and other related networks have been studied in [26].

If we add a vertex in each face of a planar graph G and then join it to all the vertices of the respective face, we get the stellation of G, denoted as $St(G)$ [23]. The dual of a planar graph G, denoted by $Du(G)$, is a graph whose vertex set is the set of faces of G, where two vertices f' and g' are joined in $Du(G)$ by an edge e' if the faces f and g share the edge e in graph G. Clearly the number of vertices of $Du(G)$ is equal to the number of faces of G and the number of edges of $Du(G)$ is equal to the number of edges of G. Since every planar graph has exactly one unbounded face, by deleting the vertex corresponding to the unbounded face in $Du(G)$, we get the bounded dual of the graph G, denoted as $Bdu(G)$. These two operations are explained in Fig. 2.

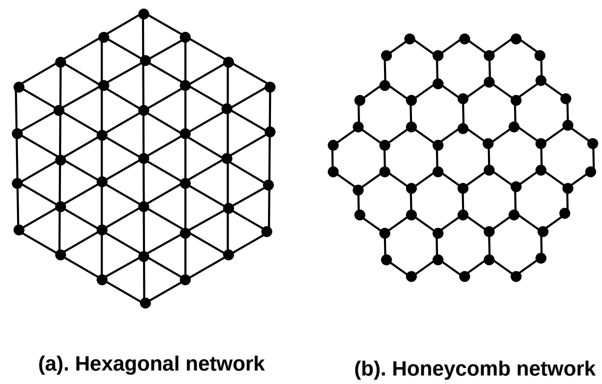

Fig. 1. (a) A hexagonal network of dimension 4; (b) A 3-dimensional honeycomb network.

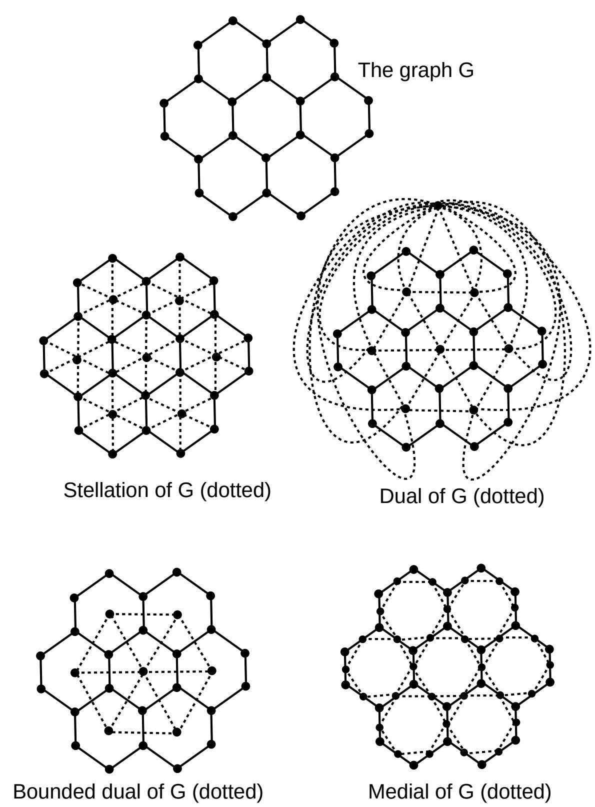

Fig. 2. Stellation, dual, bounded dual and medial of a graph G.

In this section, we derive some new classes of networks from honeycomb network by using some basic graph operations like stellation, bounded dual and medial of a graph. These operations are already defined and explained in previous section. It can easily be observed that the bounded dual of n-dimensional honeycomb networks is the n-dimensional hexagonal network. Table 1 shows derivation of honeycomb derived networks in which G is the honeycomb networks, $St(G)$ is the stellation of G and $Bdu(G)$ is the bounded dual of G. We denote n-dimensional honeycomb derived network of first type as $HcDN1(n)$ and of second type as $HcDN2(n)$.

Table 1. Derivation of honeycomb derived networks

$G\cup$	$St(G)\cup$	$Bdu(G)\cup$	$Md(G)$	Planar/Non-planar
$HcDN1$	✓	×	×	Planar
$HcDN2$	✓	✓	×	Non-planar
$HcDN3$	✓	×	✓	Non-planar
$HcDN4$	✓	✓	✓	Non-planar

Now we study the topological indices of honeycomb derived networks of first type *i.e.* $HcDN1(n)$. A 3-dimensional $HcDN1$ network is depicted in Fig. 3, in which black colored graph is 3-dimensional honeycomb network and blue colored graph is its stellation.

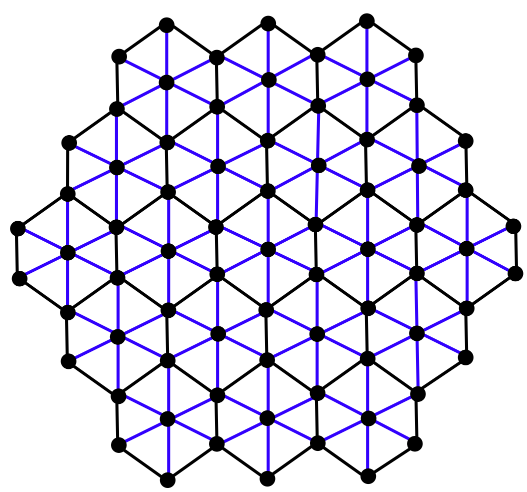

Fig. 3. An $HcDN1(n)$ network with $n = 3$.

Now we compute degree based topological indices of honeycomb derived ($HcDN1$) networks in the following theorems. First, we compute general Randić index $R_\alpha(G)$, with $\alpha = 1, -1, \frac{1}{2}, -\frac{1}{2}$.

Theorem 2.1.1. *Consider the honeycomb derived network* $HcDN1(n)$, $n \geq 3$, *then its general Randić index is equal to*

$$R_\alpha(HcDN1(n)) = \begin{cases} 972n^2 - 1224n + 414, & \alpha = 1; \\ 162n^2 + (12\sqrt{15} + 18\sqrt{2} + 18\sqrt{30} - \\ 342)n - 12\sqrt{15} - 18\sqrt{30} + 234, & \alpha = \frac{1}{2}; \\ \frac{3}{4}n^2 + \frac{3}{20}n + \frac{1}{10}, & \alpha = -1; \\ \frac{9}{2}n^2 + (\frac{4\sqrt{15}}{5} + \frac{3\sqrt{30}}{5} + \sqrt{2} - \frac{19}{2})n - \\ \frac{4\sqrt{15}}{5} - \frac{3\sqrt{30}}{5} + 7, & \alpha = -\frac{1}{2}. \end{cases}$$

Proof. Let G be the $HcDN1(n)$ network of dimension n. The number of vertices and edges in $HcDN1(n)$ are $9n^2 - 3n + 1$ and $27n^2 - 21n + 6$, respectively. There are five types of edges in $HcDN1(n)$ based on degrees of end vertices of each edge. Table 2 shows such an edge partition of $HcDN1(n)$.

Table 2. Edge partition of honeycomb derived network $HcDN1(n)$, $n \geq 3$ based on degrees of end vertices of each edge

(d_u, d_v) where $uv \in E(G)$	Number of edges
$(3, 3)$	6
$(3, 5)$	$12(n-1)$
$(3, 6)$	$6n$
$(5, 6)$	$18(n-1)$
$(6, 6)$	$27n^2 - 57n + 30$

We consider the following cases for the possible values of α.

Case 1. $\alpha = 1$

We apply the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = 1$. By using edge partition given in Table 2, we get

$$R_1(G) = 972n^2 - 1224n + 414.$$

Case 2. $\alpha = \frac{1}{2}$

We apply the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = \frac{1}{2}$. By using edge partition given in table 2, we get

$$R_{\frac{1}{2}}(G) = 162n^2 + (12\sqrt{15} + 18\sqrt{2} + 18\sqrt{30} - 342)n - 12\sqrt{15} - 18\sqrt{30} + 234.$$

Case 3. $\alpha = -1$

We apply the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = -1$. By using edge partition given in table 2, we get

$$R_{-1}(G) = \frac{3}{4}n^2 + \frac{3}{20}n + \frac{1}{10}.$$

Case 4. $\alpha = -\dfrac{1}{2}$

We apply the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = -\dfrac{1}{2}$. By using edge partition given in Table 2, we get

$$R_{-\frac{1}{2}}(G) \;=\; \frac{9}{2}n^2 + \left(\frac{4\sqrt{15}}{5} + \frac{3\sqrt{30}}{5} + \sqrt{2} - \frac{19}{2}\right)n - \frac{4\sqrt{15}}{5} - \frac{3\sqrt{30}}{5} + 7.$$

□

In the following theorem, we compute first Zagreb index of an n-dimensional honeycomb derived network of first type.

Theorem 2.1.2 *For an n-dimensional honeycomb network $HcDN1(n)$, $n \geq 3$, the first Zagreb index is equal to*

$$M_1(HcDN1(n)) = 324n^2 - 336n + 102.$$

Proof. Let G be the $HcDN1(n)$ network with $n \geq 3$. By using the formula given by equation (2) and the edge partition given in table 2, we get

$$M_1(G) \;=\; 324n^2 - 336n + 102.$$

□

Now we compute ABC index of honeycomb derived network $HcDN1(n)$.

Theorem 2.1.3 *Consider the honeycomb derived network $HcDN1(n)$, $n \geq 3$, then its ABC index is equal to*

$$ABC(HcDN1(n)) = \frac{9\sqrt{10}}{2}n^2 + \left(\frac{12\sqrt{10}}{5} + \frac{9\sqrt{30}}{5} - \frac{19\sqrt{10}}{2} + 14\right)n - \frac{12\sqrt{10}}{5} - \frac{9\sqrt{30}}{5} + 5\sqrt{10} + 4.$$

Proof. Let G be the honeycomb derived network of first type. By using the formula given by equation (4) and using edge partition given in Table 2, we get

$$ABC(G) = \frac{9\sqrt{10}}{2}n^2 + \left(\frac{12\sqrt{10}}{5} + \frac{9\sqrt{30}}{5} - \frac{19\sqrt{10}}{2} + 14\right)n - \frac{12\sqrt{10}}{5} - \frac{9\sqrt{30}}{5} + 5\sqrt{10} + 4.$$

□

In the following theorem, we compute GA index of honeycomb derived network $HcDN1(n)$.

Theorem 2.1.4 *Consider the honeycomb derived network $HcDN1(n)$, $n \geq 3$, then its GA index is equal to*

$$GA(G) = 27n^2 + \left(\frac{36\sqrt{30}}{11} + 3\sqrt{15} + 4\sqrt{2} - 57\right)n - \frac{36\sqrt{30}}{11} - 3\sqrt{15} + 36.$$

Proof. Let G be the honeycomb derived network of first type. By using the formula given by equation (5) and using the edge partition given in Table 2, we get

$$GA(G) \;=\; 27n^2 + \left(\frac{36\sqrt{30}}{11} + 3\sqrt{15} + 4\sqrt{2} - 57\right)n - \frac{36\sqrt{30}}{11} - 3\sqrt{15} + 36.$$

□

Now we compute the above discussed degree based topological indices for the second type of honeycomb derived networks, that is $HcDN2(n)$. An $HcDN2(n)$ network is defined as the union of honeycomb network, its stellation and its bounded dual as shown in Table 1. A 3-dimensional $HcDN2$ network is depicted in Fig. 4 in which black colored graph is a honeycomb network, blue colored graph is its stellation and the red color represents its bounded dual.

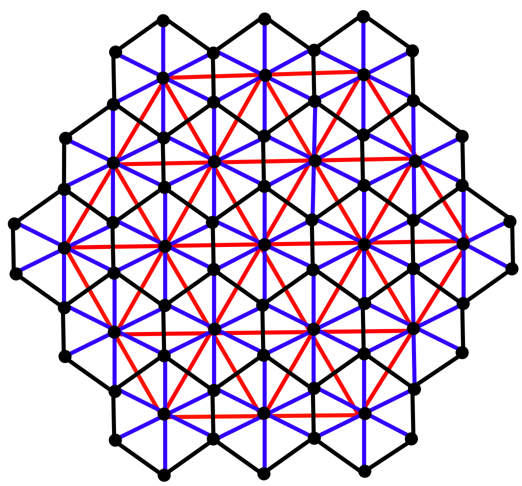

Fig. 4. An $HcDN2(n)$ network with $n = 3$.

In following theorems, we calculate the exact expressions for the general Randić index $R_\alpha(G)$ for different values of α.

Theorem 2.1.5 *Consider the honeycomb derived $HcDN2(n)$ networks with $n \geq 3$, then its general Randić index is equal to*

$$R_\alpha(HcDN2(n)) = \begin{cases} 2916n^2 - 5136n + 2310, & \alpha = 1; \\ (108\sqrt{2}+162)n^2 + (48\sqrt{15}+36\sqrt{30}- \\ 174\sqrt{2}-462)n - 84\sqrt{15}+72\sqrt{3}-66\sqrt{30}+ \\ 36\sqrt{5}-132\sqrt{2}+36\sqrt{6}+36\sqrt{10}+270, & \alpha = \frac{1}{2}; \\ \frac{9}{16}n^2 + \frac{27}{80}n + \frac{97}{1800}, & \alpha = -1; \\ \left(\frac{3\sqrt{2}}{2}+\frac{9}{4}\right)n^2 + \left(\frac{7\sqrt{15}}{5}+\frac{3\sqrt{30}}{5}-\frac{33\sqrt{2}}{10}- \right. \\ \left.\frac{113}{20}\right)n - 2\sqrt{15} + \frac{5\sqrt{3}}{3} - \sqrt{30} + \frac{4\sqrt{5}}{5} + \\ \frac{11\sqrt{2}}{10} + \frac{2\sqrt{6}}{3} + \frac{2\sqrt{10}}{5} + \frac{47}{10}, & \alpha = -\frac{1}{2}. \end{cases}$$

Proof. Let G be the $HcDN2(n)$ network of dimension n, where $n \geq 3$. The vertex and edge cardinalities of $HcDN2(n)$ are $9n^2 - 3n + 1$ and $27n^2 - 21n + 6$ respectively. There are sixteen types of edges in an $HcDN2(n)$ network, based on degrees of end vertices of each edge. Table 3 shows such an edge partition of $HcDN2(n)$.

Table 3. Edge partition of honeycomb derived network $HcDN2(n)$, $n \geq 3$ based on degrees of end vertices of each edge

(d_u, d_v) where $uv \in E(G)$	Number of edges
$(3,3)$	6
$(3,5)$	$12(n-1)$
$(3,9)$	12
$(3,10)$	$6(n-2)$
$(5,6)$	$6(n-1)$
$(5,9)$	12
$(5,10)$	$12(n-2)$
$(6,6)$	$9n^2-21n+12$
$(6,9)$	12
$(6,10)$	$18(n-2)$
$(6,12)$	$18n^2-54n+42$
$(9,10)$	12
$(9,12)$	6
$(10,10)$	$6(n-3)$
$(10,12)$	$12(n-2)$
$(12,12)$	$9n^2-33n+30$

We consider the following cases for the possible values of α.

Case 1. $\alpha = 1$

We apply the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = 1$. By using edge partition given in Table 3, we get

$$R_1(G) \;=\; 2916n^2 - 5136n + 2310.$$

Case 2. $\alpha = \frac{1}{2}$

Using the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = \frac{1}{2}$ and edge partition given in Table 3, after some simplification, we get

$$R_{\frac{1}{2}}(G) \;=\; (108\sqrt{2} + 162)n^2 + (48\sqrt{15} + 36\sqrt{30} - 174\sqrt{2} - 462)n - 84\sqrt{15} + 72\sqrt{3} - \\ -66\sqrt{30} + 36\sqrt{5} - 132\sqrt{2} + 36\sqrt{6} + 36\sqrt{10} + 270.$$

Case 3. $\alpha = -1$

The formula of $R_\alpha(G)$ is given by equation (1) for $\alpha = -1$. By using edge partition given in Table 3 and after some simplification, we get

$$R_{-1}(G) = \frac{9}{16}n^2 + \frac{27}{80}n + \frac{97}{1800}.$$

Case 4. $\alpha = -\frac{1}{2}$

Using the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = -\frac{1}{2}$ and the edge partition given in Table 3, after some simplification, we get

$$R_{-\frac{1}{2}}(G) \;=\; (\frac{3\sqrt{2}}{2} + \frac{9}{4})n^2 + (\frac{7\sqrt{15}}{5} + \frac{3\sqrt{30}}{5} - \frac{33\sqrt{2}}{10} - \frac{113}{20})n - 2\sqrt{15} + \frac{5\sqrt{3}}{3} - \\ -\sqrt{30} + \frac{4\sqrt{5}}{5} + \frac{11\sqrt{2}}{10} + \frac{2\sqrt{6}}{3} + \frac{2\sqrt{10}}{5} + \frac{47}{10}.$$

□

In the following theorem, first Zagreb index of an n-dimensional honeycomb derived network of second type is computed.

Theorem 2.1.6. *For an n-dimensional $HcDN2(n)$ network with $n \geq 3$, the first Zagreb index is equal to*

$$M_1(HcDN2(n)) = 648n^2 - 924n + 360.$$

Proof. Let G be the $HcDN2(n)$ network with $n \geq 3$. By using the formula given by equation (2) and some easy calculations, we get

$$M_1(G) = 648n^2 - 924n + 360.$$

□

Now we compute ABC index of honeycomb derived network $HcDN2(n)$.

Theorem 2.1.7. *Consider the honeycomb derived network $HcDN2(n)$, $n \geq 3$, then its ABC index is equal to*

$$\begin{aligned} ABC(HcDN2(n)) \ = \ & \left(\frac{3\sqrt{10}}{2}+\frac{3\sqrt{22}}{4}+6\sqrt{2}\right)n^2+\left(\frac{12\sqrt{10}}{5}+\frac{\sqrt{330}}{5}+\frac{3\sqrt{30}}{5}+\right.\\ & +\frac{6\sqrt{26}}{5}+\frac{9\sqrt{2}}{5}+\frac{3\sqrt{210}}{5}-\frac{7\sqrt{10}}{2}-\frac{11\sqrt{22}}{4}-\frac{12\sqrt{10}}{5}+\\ & \left.+\frac{4\sqrt{30}}{3}-\frac{2\sqrt{330}}{5}-\frac{3\sqrt{30}}{5}+\frac{8\sqrt{15}}{5}-18\sqrt{2}+2\sqrt{6}\right)n-\\ & -\frac{12\sqrt{26}}{5}+\frac{2\sqrt{78}}{3}+\frac{2\sqrt{170}}{30}-\frac{6\sqrt{210}}{5}+\frac{5\sqrt{22}}{2}-\frac{9\sqrt{2}}{5}+\frac{\sqrt{57}}{3}+\\ & +2\sqrt{10}+14\sqrt{2}-4\sqrt{6}. \end{aligned}$$

Proof. Let G be the honeycomb derived network of first type. By using the formula given by equation (4) and doing simplification, we get the following result:

$$\begin{aligned} ABC(HcDN2(n)) \ = \ & \left(\frac{3\sqrt{10}}{2}+\frac{3\sqrt{22}}{4}+6\sqrt{2}\right)n^2+\left(\frac{12\sqrt{10}}{5}+\frac{\sqrt{330}}{5}+\frac{3\sqrt{30}}{5}+\right.\\ & +\frac{6\sqrt{26}}{5}+\frac{9\sqrt{2}}{5}+\frac{3\sqrt{210}}{5}-\frac{7\sqrt{10}}{2}-\frac{11\sqrt{22}}{4}-\frac{12\sqrt{10}}{5}+\\ & \left.+\frac{4\sqrt{30}}{3}-\frac{2\sqrt{330}}{5}-\frac{3\sqrt{30}}{5}+\frac{8\sqrt{15}}{5}-18\sqrt{2}+2\sqrt{6}\right)n-\\ & -\frac{12\sqrt{26}}{5}+\frac{2\sqrt{78}}{3}+\frac{2\sqrt{170}}{30}-\frac{6\sqrt{210}}{5}+\frac{5\sqrt{22}}{2}-\frac{9\sqrt{2}}{5}+\frac{\sqrt{57}}{3}+\\ & +2\sqrt{10}+14\sqrt{2}-4\sqrt{6}. \end{aligned}$$

□

The following theorem exhibits GA index of honeycomb derived network of second version *i.e.* $HcDN2(n)$.

Theorem 2.1.8. *Consider the honeycomb derived network $HcDN2(n)$, $n \geq 3$, then its GA index is equal to*

$$\begin{aligned} GA(HcDN2(n)) \ = \ & \left(12\sqrt{2}+18\right)n^2+\left(\frac{600\sqrt{30}}{143}+\frac{9\sqrt{15}}{2}+3\sqrt{15}-28\sqrt{2}-48\right)n-\\ & -\frac{1044\sqrt{30}}{143}+\frac{36\sqrt{5}}{7}+\frac{24\sqrt{6}}{5}+\frac{72\sqrt{10}}{19}+\frac{24\sqrt{3}}{7}-3\sqrt{15}+6\sqrt{3}-\\ & -16\sqrt{2}-9\sqrt{15}+28\sqrt{2}+30. \end{aligned}$$

Proof. Let G be the honeycomb derived network of first type. By using the formula given by equation (5) and the edge partition given in Table 3, after simplification we get:

$$GA(HcDN2(n)) = \left(12\sqrt{2}+18\right)n^2 + \left(\frac{600\sqrt{30}}{143} + \frac{9\sqrt{15}}{2} + 3\sqrt{15} - 28\sqrt{2} - 48\right)n - \\ -\frac{1044\sqrt{30}}{143} + \frac{36\sqrt{5}}{7} + \frac{24\sqrt{6}}{5} + \frac{72\sqrt{10}}{19} + \frac{24\sqrt{3}}{7} - 3\sqrt{15} + 6\sqrt{3} - \\ -16\sqrt{2} - 9\sqrt{15} + 28\sqrt{2} + 30.$$

□

Now we study above discussed topological indices for third type of honeycomb derived networks, that is $HcDN3(n)$. An $HcDN3(n)$ network is defined as a union of honeycomb network, its stellation and its medial. The vertex and edge cardinalities of a $HcDN3(n)$ network are $18n^2-6n+1$ and $54n^2-42n+12$ respectively. An $HcDN3(3)$ is depicted in Fig. 5, in which green colored graph is the medial of honeycomb network.

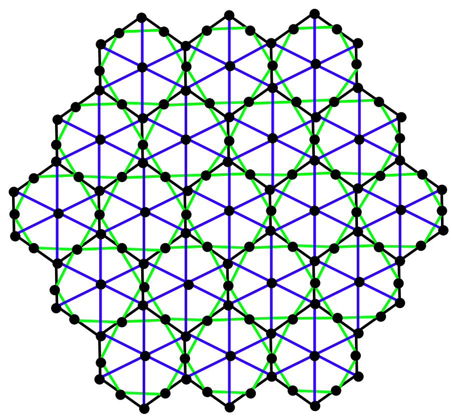

Fig. 5. An $HcDN3(n)$ network with $n = 3$.

Now we compute general Randić index for different values of α.

Theorem 2.1.9. *Consider the honeycomb derived $HcDN3(n)$ networks with $n \geq 3$, then its general Randić index is equal to:*

$$R_\alpha(HcDN3(n)) = \begin{cases} 1944n^2 - 2472n + 876, & \alpha = 1; \\ 324n^2 + (24\sqrt{3} + 18\sqrt{2} + 18\sqrt{30} + \\ 24\sqrt{5} + 24\sqrt{6} - 624)n - 24\sqrt{5} - \\ -24\sqrt{6} - 18\sqrt{30} + 324, & \alpha = \frac{1}{2}; \\ \frac{3}{2}n^2 + \frac{49}{120}n - \frac{1}{5}, & \alpha = -1; \\ 9n^2 + (\frac{6\sqrt{5}}{5} + \frac{3\sqrt{30}}{5} + 2\sqrt{3} + \sqrt{2} + \sqrt{6} - \frac{33}{2})n - \\ -\frac{6\sqrt{5}}{5} - \frac{3\sqrt{30}}{5} - \sqrt{6} - 9, & \alpha = -\frac{1}{2}. \end{cases}$$

Proof. Let G be the $HcDN3(n)$ network of dimension n. There are seven types of edges in $HcDN3(n)$ based on degrees of end vertices of each edge. Table 4 shows such an edge partition of $HcDN3(n)$.

Table 4. Edge partition of honeycomb derived network $HcDN3(n)$, $n \geq 3$ based on degrees of end vertices of each edge

(d_u, d_v) where $uv \in E(G)$	Number of edges
$(3,4)$	$12n$
$(3,6)$	$6n$
$(4,4)$	$6n$
$(4,5)$	$12(n-1)$
$(4,6)$	$12(n-1)$
$(5,6)$	$18(n-1)$
$(6,6)$	$54n^2 - 108n + 54$

We consider the following cases for the possible values of α.
Case 1. $\alpha = 1$
We apply the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = 1$. By using edge partition given in Table 4 and after simplifying, we get

$$R_1(G) = 1944n^2 - 2472n + 876.$$

Case 2. $\alpha = \frac{1}{2}$
We apply the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = \frac{1}{2}$. By using edge partition given in Table 4, some simplification gives

$$\begin{aligned} R_{\frac{1}{2}}(G) &= 324n^2 + (24\sqrt{3} + 18\sqrt{2} + 18\sqrt{30} + 24\sqrt{5} + 24\sqrt{6} - 624)n - \\ & \quad 24\sqrt{5} - 24\sqrt{6} - 18\sqrt{30} + 324. \end{aligned}$$

Case 3. $\alpha = -1$
We apply the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = -1$. By using edge partition given in Table 4, then after simplification we get

$$R_{-1}(G) = \frac{3}{2}n^2 + \frac{49}{120}n - \frac{1}{5}.$$

Case 4. $\alpha = -\frac{1}{2}$
We apply the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = -\frac{1}{2}$. By using edge partition given in Table 4, then after simplification we get

$$R_{-\frac{1}{2}}(G) = 9n^2 + \left(\frac{6\sqrt{5}}{5} + \frac{3\sqrt{30}}{5} + 2\sqrt{3} + \sqrt{2} + \sqrt{6} - \frac{33}{2}\right)n - \frac{6\sqrt{5}}{5} - \frac{3\sqrt{30}}{5} - \sqrt{6} - 9.$$

□

In the following theorem, we compute first Zagreb index of an n-dimensional honeycomb derived network of third type.

Theorem 2.1.10. *For an n-dimensional $HcDN3(n)$ network with $n \geq 3$, then first Zagreb index is equal to:*

$$M_1(HcDN3(n)) = 648n^2 - 684n + 222.$$

Proof. Let G be the $HcDN3(n)$ network with $n \geq 3$. By using the formula given by equation (2) and using edge partition given in Table 4 and after simplifications we get

$$M_1(G) = 648n^2 - 684n + 222.$$

□

Now we compute ABC index of honeycomb derived network $HcDN3(n)$.

Theorem 2.1.11. *Consider the honeycomb derived network $HcDN3(n)$, $n \geq 3$, then its ABC index is equal to:*

$$ABC(HcDN3(n)) = 9\sqrt{10}n^2 + \left(\frac{6\sqrt{35}}{5} + \frac{9\sqrt{30}}{5} + \frac{3\sqrt{6}}{2} + 2\sqrt{15} + 4\sqrt{3} + \sqrt{14} - 18\right)n - \frac{6\sqrt{35}}{5} - \frac{9\sqrt{30}}{5} + 9\sqrt{10} - 4\sqrt{3}.$$

Proof. Let G be the honeycomb derived network of third type. By using the formula given by equation (4) and by using edge partition given in Table 4 and some simplifications, we get:

$$ABC(HcDN3(n)) = 9\sqrt{10}n^2 + \left(\frac{6\sqrt{35}}{5} + \frac{9\sqrt{30}}{5} + \frac{3\sqrt{6}}{2} + 2\sqrt{15} + 4\sqrt{3} + \sqrt{14} - 18\right)n - \frac{6\sqrt{35}}{5} - \frac{9\sqrt{30}}{5} + 9\sqrt{10} - 4\sqrt{3}.$$

□

In the following theorem, we compute GA index of honeycomb derived network $HcDN3(n)$.

Theorem 2.1.12. *Consider the honeycomb derived network $HcDN3(n)$, $n \geq 3$, then its GA index is equal to:*

$$GA(G) = 54n^2 + \left(\frac{48\sqrt{3}}{7} + \frac{24\sqrt{6}}{5} + \frac{16\sqrt{5}}{3} + \frac{36\sqrt{30}}{11} + 4\sqrt{2} - 102\right)n - \frac{16\sqrt{5}}{3} - \frac{24\sqrt{6}}{5} - \frac{36\sqrt{30}}{11} + 54.$$

Proof. Let G be the honeycomb derived network of third type. By using the formula given by equation (1) and using edge partition given in Table 4 and doing some simplifications, we get:

$$GA(G) = 54n^2 + \left(\frac{48\sqrt{3}}{7} + \frac{24\sqrt{6}}{5} + \frac{16\sqrt{5}}{3} + \frac{36\sqrt{30}}{11} + 4\sqrt{2} - 102\right)n - \\ -\frac{16\sqrt{5}}{3} - \frac{24\sqrt{6}}{5} - \frac{36\sqrt{30}}{11} + 54.$$

□

Now we study the topological indices of the fourth type of honeycomb derived networks, that is $HcDN4(n)$. An $HcDN4(n)$ network is defined as the union of honeycomb network, its stellation, its medial and its bounded dual, as shown in Table 1. A 3-dimensional $HcDN4$ network is depicted in Fig. 6, in which black colored graph is honeycomb network, blue colored graph is its stellation, green color is its medial and red color represents its bounded dual. This network has $18n^2 - 6n + 1$ vertices and $63n^2 - 57n + 18$ edges.

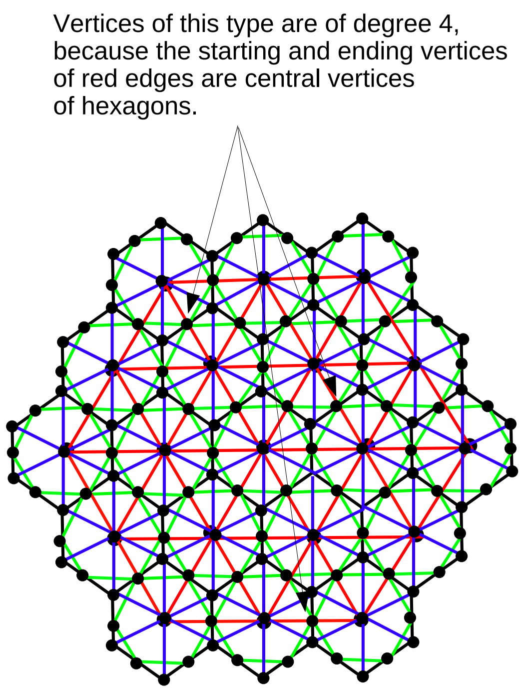

Fig. 6. An $HcDN4(n)$ network with $n = 3$.

In the next theorem, we compute exact expressions for the general Randić index $R_\alpha(G)$, for $\alpha = 1, \frac{1}{2}, -1, -\frac{1}{2}$.

Theorem 2.1.13. *Consider the honeycomb derived network* $HcDN4(n)$ *with* $n \geq 3$*, then its general Randić index is equal to*:

$$R_\alpha(HcDN4(n)) = \begin{cases} 3888n^2 - 6384n + 2772, & \alpha = 1; \\ (108\sqrt{2}+324)n^2 + (24\sqrt{3}+36\sqrt{30}+24\sqrt{5}+24\sqrt{6}+ \\ +36\sqrt{15}-264\sqrt{2}-744)n + 72\sqrt{3}-66\sqrt{30}-24\sqrt{5}+ \\ +36\sqrt{5}-120\sqrt{2}-72\sqrt{15}+252\sqrt{2}+36\sqrt{10}+396, & \alpha = \frac{1}{2}; \\ \frac{21}{16}n^2 + \frac{143}{240}n - \frac{443}{1800}, & \alpha = -1; \\ \left(\frac{3\sqrt{2}}{2}+\frac{27}{4}\right)n^2 + \left(\frac{3\sqrt{15}}{5}+\frac{6\sqrt{5}}{5}-\frac{33\sqrt{2}}{10}+\frac{3\sqrt{30}}{5}+ \right. \\ \left. +2\sqrt{3}+\sqrt{6}-\frac{253}{20}\right)n + \frac{5\sqrt{3}}{3} - \sqrt{30} - \frac{2\sqrt{5}}{5} - \\ -\frac{11\sqrt{2}}{10} + \frac{2\sqrt{10}}{5} - \frac{6\sqrt{15}}{5} + \frac{67}{10}, & \alpha = -\frac{1}{2}. \end{cases}$$

Proof. Let G be the $HcDN4(n)$ network of dimension n with $n \geq 3$. There are eighteen types of edges in $HcDN4(n)$, based on the degrees of end vertices of each edge which make a partition of edge set of G. Table 5 shows such an edge partition of $HcDN4(n)$.

Table 5. Edge partition of honeycomb derived network $HcDN4(n)$, $n \geq 3$ based on degrees of end vertices of each edge.

(d_u, d_v) where $uv \in E(G)$	Number of edges
$(3,4)$	$12n$
$(3,9)$	12
$(3,10)$	$6(n-2)$
$(4,4)$	$6n$
$(4,5)$	$12(n-1)$
$(4,6)$	$12(n-1)$
$(5,6)$	$6(n-1)$
$(5,9)$	12
$(5,10)$	$12(n-2)$
$(6,6)$	$36n^2-72n+36$
$(6,9)$	12
$(6,10)$	$18(n-2)$
$(6,12)$	$18n^2-54n+42$
$(9,10)$	12
$(9,12)$	6
$(10,10)$	$6(n-3)$
$(10,12)$	$12(n-2)$
$(12,12)$	$9n^2-33n+30$

We consider the following cases for the possible values of α.

Case 1. $\alpha = 1$

Using the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = 1$ and the edge partition given in Table 5, we get

$$R_1(G) = 3888n^2 - 6384n + 2772.$$

Case 2. $\alpha = \frac{1}{2}$.

Using the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = \frac{1}{2}$ and using edge partition given in Table 5, we get

$$\begin{aligned} R_{\frac{1}{2}}(G) \;=\; & \left(\frac{3\sqrt{2}}{2} + \frac{27}{4}\right)n^2 + \left(\frac{3\sqrt{15}}{5} + \frac{6\sqrt{5}}{5} - \frac{33\sqrt{2}}{10} + \frac{3\sqrt{30}}{5} + 2\sqrt{3} + \sqrt{6} - \right. \\ & \left. - \frac{253}{20}\right)n + \frac{5\sqrt{3}}{3} - \sqrt{30} - \frac{2\sqrt{5}}{5} - \frac{11\sqrt{2}}{10} + \frac{2\sqrt{10}}{5} - \frac{6\sqrt{15}}{5} + \frac{67}{10}. \end{aligned}$$

Case 3. $\alpha = -1$.
Applying the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = -1$ and by using edge partition given in Table 5, we get

$$R_{-1}(G) = \frac{21}{16}n^2 + \frac{143}{240}n - \frac{443}{1800}.$$

Case 4. $\alpha = -\frac{1}{2}$.

Applying the formula of $R_\alpha(G)$ given by equation (1) for $\alpha = -\frac{1}{2}$ and using edge partition given in Table 5, we get

$$\begin{aligned} R_{-\frac{1}{2}}(G) \;=\; & \left(\frac{3\sqrt{2}}{2} + \frac{9}{4}\right)n^2 + \left(\frac{7\sqrt{15}}{5} + \frac{3\sqrt{30}}{5} - \frac{33\sqrt{2}}{10} - \frac{113}{20}\right)n - 2\sqrt{15} + \frac{5\sqrt{3}}{3} - \\ & - \sqrt{30} + \frac{4\sqrt{5}}{5} + \frac{11\sqrt{2}}{10} + \frac{2\sqrt{6}}{3} + \frac{2\sqrt{10}}{5} + \frac{47}{10}. \end{aligned}$$

□

In the following theorem, we compute first Zagreb index of an n-dimensional honeycomb derived network of fourth type.

Theorem 2.1.14. *For an n-dimensional $HcDN4(n)$ network with $n \geq 3$, the first Zagreb index is equal to:*

$$M_1(HcDN4(n)) = 972n^2 - 1272n + 480.$$

Proof. Let G be the $HcDN4(n)$ network with $n \geq 3$. By using the formula given by equation (2) and the edge partition from Table 5, we get:

$$M_1(G) = 972n^2 - 1272n + 480.$$

□

Now we compute ABC index of honeycomb derived network $HcDN4(n)$.

Theorem 2.1.15. *Consider the honeycomb derived network $HcDN4(n)$, $n \geq 3$, then its ABC index is equal to:*

$$\begin{aligned} ABC(HcDN4(n)) \; = \; & \left(\frac{3\sqrt{22}}{4} + 6\sqrt{2} + 6\sqrt{10}\right)n^2 + \left(\frac{7\sqrt{6}}{2} + \frac{6\sqrt{35}}{5} + \frac{3\sqrt{30}}{5} + \right. \\ & + \frac{6\sqrt{26}}{5} + \frac{3\sqrt{210}}{5} - \frac{81\sqrt{2}}{5} - \frac{11\sqrt{22}}{4} + \frac{\sqrt{330}}{5} + 2\sqrt{15} + \\ & \left. +4\sqrt{3} - 12\sqrt{10}\right)n + \frac{4\sqrt{30}}{3} - \frac{2\sqrt{330}}{5} - \frac{6\sqrt{35}}{5} - \frac{3\sqrt{30}}{5} + \\ & + \frac{8\sqrt{15}}{5} - \frac{12\sqrt{26}}{5} + \frac{2\sqrt{78}}{3} + \frac{2\sqrt{170}}{5} - \frac{6\sqrt{210}}{5} + \frac{5\sqrt{22}}{2} + \\ & + \frac{61\sqrt{2}}{5} + \frac{\sqrt{57}}{3} - 4\sqrt{3} + 6\sqrt{10} - 4\sqrt{6}. \end{aligned}$$

Proof. Let G be the honeycomb derived network of fourth type. By using the formula given by equation (4) and using edge partition given in Table 5, we get:

$$\begin{aligned} ABC(HcDN4(n)) \; = \; & \left(\frac{3\sqrt{22}}{4} + 6\sqrt{2} + 6\sqrt{10}\right)n^2 + \left(\frac{7\sqrt{6}}{2} + \frac{6\sqrt{35}}{5} + \frac{3\sqrt{30}}{5} + \right. \\ & + \frac{6\sqrt{26}}{5} + \frac{3\sqrt{210}}{5} - \frac{81\sqrt{2}}{5} - \frac{11\sqrt{22}}{4} + \frac{\sqrt{330}}{5} + 2\sqrt{15} + \\ & \left. +4\sqrt{3} - 12\sqrt{10}\right)n + \frac{4\sqrt{30}}{3} - \frac{2\sqrt{330}}{5} - \frac{6\sqrt{35}}{5} - \frac{3\sqrt{30}}{5} + \\ & + \frac{8\sqrt{15}}{5} - \frac{12\sqrt{26}}{5} + \frac{2\sqrt{78}}{3} + \frac{2\sqrt{170}}{5} - \frac{6\sqrt{210}}{5} + \frac{5\sqrt{22}}{2} + \\ & + \frac{61\sqrt{2}}{5} + \frac{\sqrt{57}}{3} - 4\sqrt{3} + 6\sqrt{10} - 4\sqrt{6}. \end{aligned}$$

□

In the following theorem, we compute GA index of honeycomb derived network $HcDN4(n)$.

Theorem 2.1.16. *Consider the honeycomb derived network $HcDN4(n)$, $n \geq 3$, then its GA index is equal to:*

$$\begin{aligned} GA(HcDN4(n)) \; = \; & \left(12\sqrt{2} + 45\right)n^2 + \left(\frac{48\sqrt{3}}{7} + \frac{600\sqrt{30}}{143} + \frac{16\sqrt{5}}{3} + \frac{24\sqrt{6}}{5} + \right. \\ & \left. + \frac{9\sqrt{15}}{2} - 28\sqrt{2} - 93\right)n - \frac{1044\sqrt{30}}{143} - \frac{4\sqrt{5}}{21} + \frac{72\sqrt{10}}{19} + \\ & + \frac{66\sqrt{3}}{7} - 9\sqrt{15} + 12\sqrt{2} + 48. \end{aligned}$$

Proof. Let G be the honeycomb derived network of fourth type. By using the formula given by equation (5) and using edge partition given in Table 5, we get:

$$GA(G) = 12n\left(\frac{2\sqrt{3\times 4}}{3+4}\right)+12\left(\frac{2\sqrt{3\times 9}}{3+9}\right)+(6n-12)\left(\frac{2\sqrt{3\times 10}}{3+10}\right)+6n\left(\frac{2\sqrt{4\times 4}}{4+4}\right)+ (12n-12)\left(\frac{2\sqrt{4\times 5}}{4+5}\right)+(12n-12)\left(\frac{2\sqrt{4\times 6}}{4+6}\right)+(6n-6)\left(\frac{2\sqrt{5\times 6}}{5+6}\right)+12\left(\frac{2\sqrt{5\times 9}}{5+9}\right)+ (12n-24)\left(\frac{2\sqrt{5\times 10}}{5+10}\right)+(36n^2-72n+36)\left(\frac{2\sqrt{6\times 6}}{6+6}\right)+12\left(\frac{2\sqrt{6\times 9}}{6+9}\right)+(18n- -36)\left(\frac{2\sqrt{6\times 10}}{6+10}\right)+(18n^2-54n+42)\left(\frac{2\sqrt{6\times 12}}{6+12}\right)+12\left(\frac{2\sqrt{9\times 10}}{9+10}\right)+6\left(\frac{2\sqrt{9\times 12}}{9+12}\right)+ (6n-18)\left(\frac{2\sqrt{10\times 10}}{10+10}\right)+(12n-24)\left(\frac{2\sqrt{10\times 12}}{10+12}\right)+(9n^2-33n+30)\left(\frac{2\sqrt{12\times 12}}{12+12}\right).$$

After simplification, it gives

$$\begin{aligned} GA(HcDN4(n)) \;=\; & \left(12\sqrt{2}+45\right)n^2+\left(\frac{48\sqrt{3}}{7}+\frac{600\sqrt{30}}{143}+\frac{16\sqrt{5}}{3}+\frac{24\sqrt{6}}{5}+\right. \\ & \left.+\frac{9\sqrt{15}}{2}-28\sqrt{2}-93\right)n-\frac{1044\sqrt{30}}{143}-\frac{4\sqrt{5}}{21}+\frac{72\sqrt{10}}{19}+ \\ & +\frac{66\sqrt{3}}{7}-9\sqrt{15}+12\sqrt{2}+48. \end{aligned}$$

□

3. Conclusion and general remarks

In this paper, certain degree based topological indices, namely general Randić index (R_α), atom-bond connectivity index (ABC), geometric-arithmetic index (GA) and first Zagreb index (M_1) for hexagonal and honeycomb derived networks were studied for the first time. We defined some new classes of networks derived from the already existing hexagonal and honeycomb networks by using some basic graph operations. We computed analytical close formulas of the above mentioned degree based topological indices for these derived networks. These results provide a basis to understand deep topology of these important networks. The reader is encouraged to design some new architectures/networks and study their topological indices to understand their topologies.

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